PHY322 S26
Discussion 11
TA: Junqi Zhang

The main goal of this discussion is to help you prepare for midterm 2.

Electrostatics and magnetostatics

The knowledge about static electromagnetism we acquired can be summarized as follows.

	Electrostatics	**Magnetostatics**
Experimental laws	Coulomb's law $\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int d\tau' \frac{\rho(\vec{r}')(\vec{r}-\vec{r}')}{\vert\vec{r}-\vec{r}'\vert^3}$	The Biot-Savart law $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int d\tau' \frac{\vec{J}(\vec{r}')\times(\vec{r}-\vec{r}')}{\vert\vec{r}-\vec{r}'\vert^3}$
Take del of exp. laws	$\begin{cases}\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0 \quad \text{(Gauss's law)} \\ \vec{\nabla}\times\vec{E} = 0\end{cases}$	$\begin{cases}\vec{\nabla}\times\vec{B} = \mu_0\vec{J} \quad \text{(Ampere's law)} \\ \vec{\nabla}\cdot\vec{B} = 0\end{cases}$
Potential & symmetry	$\vec{\nabla}\times\vec{E} = 0 \rightarrow \vec{E} = -\vec{\nabla}V$ $V \rightarrow V + \text{constant}$	$\vec{\nabla}\cdot\vec{B} = 0 \rightarrow \vec{B} = \vec{\nabla}\times\vec{A}$ $\vec{A} \rightarrow \vec{A} + \vec{\nabla}\lambda$
Potential PDE	$\nabla^2 V = -\frac{\rho}{\epsilon_0}$ (Poisson's equation)	$\nabla^2\vec{A} = -\mu_0\vec{J}$ (in Coulomb gauge $\vec{\nabla}\cdot\vec{A} = 0$)
Potential solution	$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int d\tau' \frac{\rho(\vec{r}')}{\vert\vec{r}-\vec{r}'\vert}$	$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int d\tau' \frac{\vec{J}(\vec{r}')}{\vert\vec{r}-\vec{r}'\vert}$
Dipole moment	$\vec{p} = \int d\tau' \vec{r}' \rho(\vec{r}')$	$\vec{m} = I\int_\Sigma d\vec{a}'$
Dipole potential (with dipole at the origin)	$V_{\text{dipole}}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$	$\vec{A}_{\text{dipole}}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{\vec{m}\times\hat{r}}{r^2}$

Table 1: Parallels between electrostatics and magnetostatics in vacuum.

	Electrostatics	**Magnetostatics**
Macroscopic response	$\vec{P} \equiv$ electric dipole moment density (polarization)	$\vec{M} \equiv$ magnetic dipole moment density (magnetization)
Potential induced by dipole	$\begin{cases} V_{\text{polarization}}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\left(\int_{\mathcal{V}} d\tau' \frac{-\vec{\nabla}'\cdot\vec{P}(\vec{r'})}{\lvert\vec{r}-\vec{r'}\rvert} + \oint_{\partial\mathcal{V}} da' \frac{\vec{P}(\vec{r'})\cdot\hat{n}}{\lvert\vec{r}-\vec{r'}\rvert}\right) \\ \vec{A}_{\text{magnetization}}(\vec{r}) = \frac{\mu_0}{4\pi}\left(\int_{\mathcal{V}} d\tau' \frac{\vec{\nabla}'\times\vec{M}(\vec{r'})}{\lvert\vec{r}-\vec{r'}\rvert} + \oint_{\partial\mathcal{V}} da' \frac{\vec{M}(\vec{r'})\times\hat{n'}}{\lvert\vec{r}-\vec{r'}\rvert}\right) \end{cases}$	
Bound density	$\rho_b \equiv -\vec{\nabla}\cdot\vec{P}, \quad \sigma_b \equiv \vec{P}\cdot\hat{n}$	$\vec{J}_b \equiv \vec{\nabla}\times\vec{M}, \quad \vec{K}_b \equiv \vec{M}\times\hat{n}$
Free density	$\rho_f \equiv \rho - \rho_b$	$\vec{J}_f \equiv \vec{J} - \vec{J}_b$
Gauss's law and Ampère's law in matter	$\begin{cases} \vec{\nabla}\cdot\vec{D} = \rho_f \\ \vec{D} \equiv \epsilon_0\vec{E} + \vec{P} \end{cases}$	$\begin{cases} \vec{\nabla}\times\vec{H} = \vec{J}_f \\ \vec{H} \equiv \vec{B}/\mu_0 - \vec{M} \end{cases}$
Linear medium	$\begin{cases} \vec{P} = \epsilon_0\chi_e\vec{E} \\ \vec{D} = \epsilon\vec{E} \\ \epsilon \equiv \epsilon_0(1+\chi_e) \end{cases}$	$\begin{cases} \vec{M} = \chi_m\vec{H} \\ \vec{B} = \mu\vec{H} \\ \mu \equiv \mu_0(1+\chi_m) \end{cases}$

Table 2: Parallels between electrostatics and magnetostatics in matter.

Based on the fundamental expressions above, we can derive a set of boundary conditions for static electric and magnetic fields in matter.

- Boundary conditions in dielectrics. Using a "pill-box" type of argument, we find
  - Normal component
$$\vec{\nabla}\cdot\vec{D} = \rho_f \quad \Rightarrow \quad D^{\perp}(+) - D^{\perp}(-) = \sigma_f\,. \tag{1}$$
  - Parallel component
$$\vec{\nabla}\times\vec{E} = 0 \quad \Rightarrow \quad E^{\parallel}(+) - E^{\parallel}(-) = 0\,. \tag{2}$$
- Boundary conditions for magnetic fields in matter
  - Normal component
$$\vec{\nabla}\cdot\vec{B} = 0 \quad \Rightarrow \quad B_{\perp}(+) - B_{\perp}(-) = 0\,. \tag{3}$$
  - Parallel component
$$\vec{\nabla}\times\vec{H} = \vec{J}_f \quad \Rightarrow \quad \vec{H}_{\parallel}(+) - \vec{H}_{\parallel}(-) = \vec{K}_f\times\hat{n}\,. \tag{4}$$

Apart from these, the general solution

$$V(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta) \tag{5}$$

to Laplace's equation $\nabla^2 V = 0$ in spherical coordinates with azimuthal symmetry is worth reviewing carefully.

Ideally, with all the expressions above, one should be able to solve any problem in Griffiths' textbook. In practice, the typical recipe is

- Analyze symmetry
- Solve the field/potential via various techniques (e.g. integrated Gauss's/Ampère's law, Poisson's equation).
- **Double check your final answer** (e.g. taking limits, checking dimension).

Electrodynamics

Midterm 2 may also involve basic dynamics of the electric and magnetic fields, specifically Faraday's law. The key idea is that a changing magnetic field induces an electric field.

- In differential form, this is Faraday's law

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}\,. \tag{6}$$

- In integral form

$$\oint_{\partial\Sigma} d\vec{l} \cdot \vec{E} = -\frac{d\Phi}{dt}, \quad \Phi \equiv \int_{\Sigma} d\vec{a} \cdot \vec{B}\,. \tag{7}$$

- Direction of $\vec{E}$ is such that if there were a wire on $\partial\Sigma$, the induced current would oppose the change in Φ (Lenz's law).

Midterm 2

Since you have access to practice midterm 2 solutions, I will not talk about them here. Here are two preparation tips:

- Prioritize reviewing your homework. Ideally, you should be able to solve every problem independently, without relying on any references.
- Review your lecture notes and work through the examples again from scratch.

Good luck :)